Lösningen till EMF 15 Tentamen i Elektromagnetism

Fall för E3 2003-08-25 EP

1/ Se Chang eller föreläsningsanteckningen

2/ a/ Q enda fria laddningen. Sfärisk symmetri. Gauss lag

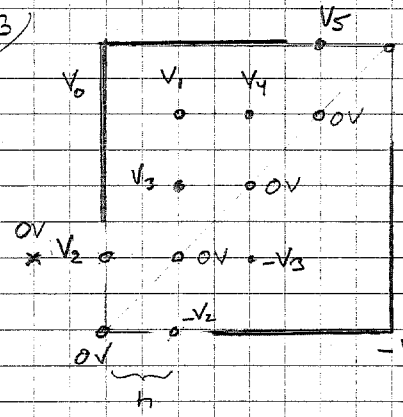
för D: $\oint D \cdot d\vec{f} = D_R 4\pi R^2 = Q_{fri\ innan} = Q$

$\Rightarrow \underline{D = \hat{R} \frac{Q}{4\pi R^2}}$ för $R > a$ $\quad D = 0$ $R < a$

$\Rightarrow \underline{E_1 = 0}$ $R < a$; $E_2 = E_4 = \underline{\frac{D}{\varepsilon_0}}$ $a < R < b$, $R > c$; $E_3 = \underline{\frac{D}{\varepsilon_r \varepsilon_0}}$ $b < R < c$

b/ $V_a - V_\infty = \int_a^\infty E \cdot dR = \underline{\frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right) + \frac{Q}{4\pi\varepsilon_r\varepsilon_0}\left(\frac{1}{b} - \frac{1}{c}\right) + \frac{Q}{4\pi\varepsilon_0 c}}$

3/



a/ Symmetri $V_4 = V_3$; $V_5 = V_2$

$$\begin{cases} V_1 = \frac{1}{4}(V_0 + V_0 + V_3 + \underbrace{V_4}_{V_3}) = \frac{1}{2}(V_0 + V_3) \\ V_3 = \frac{1}{4}(V_0 + 0 + 0 + V_1) = \frac{1}{2}(V_0 + V_1) \\ V_2 = \frac{1}{4}(V_0 + 0 + 0 + 0) = \frac{1}{4}V_0 \end{cases}$$

$\Rightarrow \underline{V_1 = \frac{5V_0}{7}}$; $\underline{V_2 = \frac{1}{4}V_0}$; $\underline{V_3 = \frac{3}{7}V_0}$

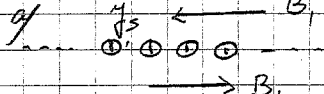
b/ Tecknar skönmen i t.ex. genom diagonalen

$E_2 = \frac{V_2 - (-V_2)}{h\sqrt{2}}$; $E_3 = \frac{V_3 - (-V_3)}{h\sqrt{2}}$; $j = \sigma E$; $i = \sum j_k S_k =$

$= dh\sqrt{2}\,\sigma\left[\frac{2V_2}{h\sqrt{2}} + \frac{2V_3}{h\sqrt{2}}\right] \cdot \underbrace{2}_{\text{pga symmetri}} = \cdots = \frac{19}{7} V_0 \sigma d$

$\Rightarrow \underline{R = \frac{2V_0}{i} = \frac{14}{19\sigma d} = 0{,}737/\sigma d}$

4/ a/

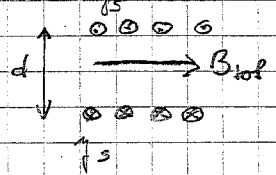


EH ∞ vett vand. Pga symmetri

B enl hg. Amperes lag $\oint B \cdot dl =$

$= B_1 \cdot b + B_1 \cdot b = \mu_0 i_{omslutet} = \mu_0 j_s b \Rightarrow B_1 = \underline{\frac{\mu_0 j_s}{2}}$; $B_{tot} = 2B_1$ mellan vandan

b/



$\Phi = \int B \cdot d\vec{f} = B_{tot}\, dl$;

induktans $L = \frac{\Phi}{i} = \frac{B_{tot}\, dl}{j_s b} = \underline{\frac{\mu_0 j_s\, dl}{j_s b}}$

$\Rightarrow \underline{L_c = \mu_0 d / b}$

5) $\bar{E}(0) = \hat{x} E_0 e^{j\pi/6}$; $\bar{E}(z) = \bar{E}(0) e^{-j\beta z} = \hat{x} E_0 e^{-j\beta z + j\pi/6}$

$\Rightarrow \bar{E}(z,t) = \hat{x} E_0 \cos(\omega t - \beta z + \pi/6)$ där $\omega = 2\pi \cdot 3 \cdot 10^8$

och $\beta = \omega\sqrt{\mu\varepsilon} = \omega\sqrt{\mu_0 \varepsilon} = \omega \cdot 3\sqrt{\mu_0\varepsilon_0} = 3\dfrac{\omega}{c_0} = \dfrac{3 \cdot 2\pi \cdot 3 \cdot 10^8}{3 \cdot 10^8} = 6\pi$

$\bar{H} = ?$ $\bar{H} = \dfrac{1}{Z}\hat{k} \times \bar{E} = \dfrac{1}{Z}\hat{y} E = \hat{y}\dfrac{E_0}{Z} e^{-j\beta z + j\pi/6}$ ($\hat{k} = \hat{z}$)

där $Z = \sqrt{\dfrac{j\omega\mu}{\sigma + j\omega\varepsilon}} = \sqrt{\dfrac{\mu_0}{\varepsilon}} = \dfrac{1}{3} Z_0$ rent reell

$\Rightarrow \bar{H}(z,t) = \hat{y}\dfrac{3E_0}{Z_0}\cos(\omega t - \beta z + \pi/6)$

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6) Reflekterad effekt $R = |\Gamma_{12}| = 0{,}999$ $R_S = \dfrac{1}{\sigma_2\delta_2} = ?$

Mycket reflekteras $\Rightarrow \sigma_2 \gg \omega\varepsilon_2$ skineffekt, Approximera!

Teckna Γ_{12}! $Z_1 = Z_0$; $Z_2 = \sqrt{\dfrac{j\omega\mu_0}{\sigma_2 + j\omega\varepsilon_2}} = (1+j)\sqrt{\dfrac{\omega\mu_0}{2\sigma_2}} = (1+j)\dfrac{1}{\sigma_2\delta_2}$ (försumbart $j\omega\varepsilon_2$)

Inträngningsdjup

$\delta_2 = \sqrt{\dfrac{2}{\omega\sigma_2\mu_0}}$ $= (1+j)R_s$

refl koeff $\Gamma_{12} = \dfrac{Z_2 - Z_0}{Z_2 + Z_0} = \dfrac{Z_2/Z_0 - 1}{Z_2/Z_0 + 1} \approx -\left(1 - \dfrac{2Z_2}{Z_0}\right) =$

$= -\left[1 - 2(1+j)\dfrac{R_s}{Z_0}\right]$ $R_s \ll Z_0$

$|\Gamma_{12}|^2 = 1 - 2 \cdot \dfrac{2R_s}{Z_0} = 1 - 0{,}001 \Rightarrow R_S = \dfrac{10^{-3} \cdot Z_0}{4} = 30\pi \cdot 10^{-3} =$

$= 94{,}2 \cdot 10^{-3}\ \Omega$