Lösningar till tentamen i Elektromagnetiska fält för E3 den 26/4 2003

EP

1) Se Cheng eller E-fält i sammandrag

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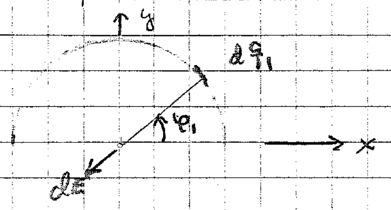
2) E från övre halvcirkeln



$dq_1 = \rho_\ell \, a \, d\varphi_1$ ger dE enl fig

$dE = \frac{dq_1}{4\pi\varepsilon_0 a^2}$; $dE_x = -dE\cos\varphi_1$

$dE_y = -dE\sin\varphi_1$; $E_x = 0$ pga symmetri

$$E_y = -\frac{\rho_\ell a}{4\pi\varepsilon_0 a^2}\int_{\varphi_1=0}^{\pi}\sin\varphi_1 \, d\varphi_1 = \dots = -\frac{\rho_\ell}{2\pi\varepsilon_0 a}$$

$-\rho_\ell$ ger lika stort bidrag $\Rightarrow$ $\mathbf{E}(0,0,0) = -\frac{\rho_\ell}{\pi\varepsilon_0 a}\hat{y}$

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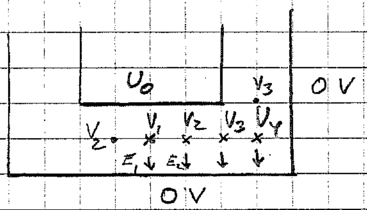
3)



a/

$$\begin{cases} V_1 = \frac{1}{4}(U_0 + V_2 + 0 + V_2) & (1) \\ V_2 = \frac{1}{4}(U_0 + V_1 + 0 + V_3) & (2) \\ V_3 = \frac{1}{4}(U_0 + V_2 + 0 + V_4) & (3) \\ V_4 = \frac{1}{4}(V_3 + V_3 + 0 + 0) & (4) \end{cases}$$

c/ lösn (1)–(4)

$$\begin{cases} V_1 = 0{,}4889\, U_0 \\ V_2 = 0{,}4778\, U_0 \\ V_3 = 0{,}4222\, U_0 \\ V_4 = 0{,}2111\, U_0 \end{cases}$$

b) Beräkna strömmen, i, genom yta mellan V_1 V_2 – och $V=0$.

$E_1 = \frac{V_1 - 0}{h}$; $E_2 = \frac{V_2 - 0}{h}$ ... ; $J = \sigma E$ $\quad i/8 = \int \mathbf{J}\cdot d\mathbf{s} =$

$= \sigma\left[E_1 \cdot \frac{h}{2} l + E_2 h l + \dots E_4 h l\right] = \sigma l \left[\frac{V_1}{2} + V_2 + V_3 + V_4\right]$;

$\Rightarrow R_\ell = \frac{U_0}{i \cdot l} = \frac{U_0}{8\sigma\left(V_1/2 + V_2 + V_3 + V_4\right)}$

c/ forts med lösn $V_1 = \dots$; $R_\ell = \frac{1}{10{,}84\sigma} = 922\ \Omega/\text{m}$ jfr dator $977\ \Omega/\text{m}$

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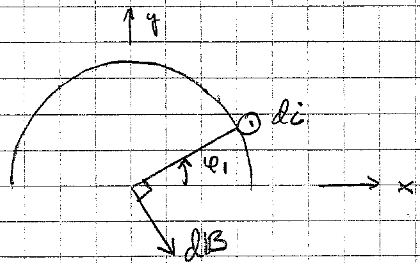
4)



dela upp strömfördelningen i tunna långa raka strömmar $di = \frac{i}{\pi a} \cdot a \, d\varphi_1$

di ger dB enl. fig

$dB = \frac{\mu_0 \, di}{2\pi a}$; $dB_x = dB \sin\varphi_1$

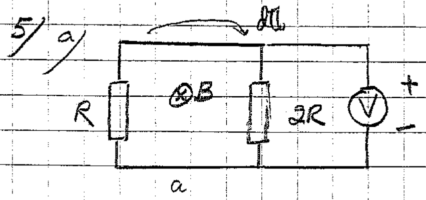
$dB_y = -dB\cos\varphi_1$ $\qquad B_y = 0$ pga symmetri

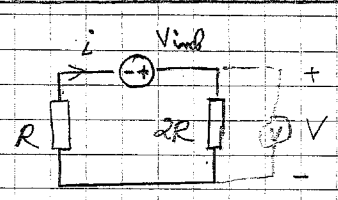
$$B_x = \frac{\mu_0 i}{2\pi^2 a}\int_0^{\pi}\sin\varphi_1 \, d\varphi_1 = \frac{\mu_0 i}{\pi^2 a}$$

$\mathbf{B} = \hat{x}\frac{\mu_0 i}{\pi^2 a}$

Lösningen till Elmagn. fält 26/4 2003 forts.

5/ a)





Inducerad spänning i slingan

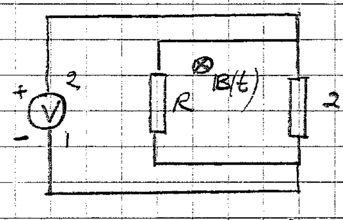
$\phi = B(t)\cdot a^2$

$V_{ind} = -\frac{d\phi}{dt} = \omega B_0 a^2 \sin\omega t$

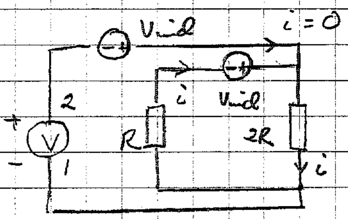
$B = B_0 \cos\omega t$; dS vald som $B \Rightarrow dl$ enl fig och −+ på V_{ind} enl fig. Voltmetern mäter $V = 2Ri = 2R\frac{V_{ind}}{R+2R} = \frac{2}{3}V_{ind}$

$V = \frac{2}{3}\omega B_0 a^2 \sin\omega t$. Om V visar effektivvärde: $V_{eff} = \frac{2}{\sqrt{2}\,3}\omega B_0 a^2$

b) Med voltmetern på andra sidan om kretsen



Inducerad spänning enl a/ men vi får V_{ind} i voltmeterkretsen också. Samma flöde omslutet ⇒ ekvivalent krets enl fig



KVL: $-V - V_{ind} + 2Ri = 0$

KVL: $-V_{ind} + 2Ri + Ri = 0$

$\Rightarrow V = -\frac{1}{3}V_{ind} = -\frac{1}{3}\omega B_0 a^2 \sin\omega t$

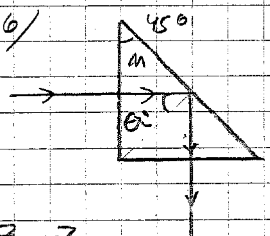
eller $V_{eff} = \frac{1}{\sqrt{2}\cdot 3}\omega B_0 a^2$

{Kan också tecknas som att V mäter (− tecken pga referensen i vår fig)

$V = -\int_1^2 E\cdot dl = -\left[\oint E\cdot dl - \int_{2\,\text{via resistansen}}^{1} E\cdot dl\right] = -\left[\underbrace{\oint E\cdot dl}_{V_{ind}} - 2Ri\right] =$

$= -V_{ind} + \frac{2}{3}V_{ind} = -\frac{1}{3}V_{ind}$ }

6/



$\theta_i = 45°$ Koll om det blir totalreflexion inne i prismat: $\theta_i > \theta_c$?

$\sin\theta_c = \frac{1}{n} = \frac{2}{3} \Rightarrow \theta_c = 41{,}8°$ ok totalrefl.

$Z_1 = Z_0$

$Z_2 = \frac{1}{\sqrt{\varepsilon_r}}Z_0 = \frac{1}{n}Z_0$

1:a genomgången $t_{12} = \frac{2Z_2}{Z_2+Z_0} = \frac{2\frac{1}{n}Z_0}{\frac{1}{n}Z_0+Z_0} = 0{,}8$

2:a genomgången $t_{21} = \frac{2Z_0}{Z_0+Z_2} = 1{,}2$

[Fasförskjutningen vid totalreflexionen ej intressant här]

$\Rightarrow \left|\frac{E_t}{E_i}\right| = |t_{12}\cdot t_{21}| = 0{,}8\cdot 1{,}2 = 0{,}96$