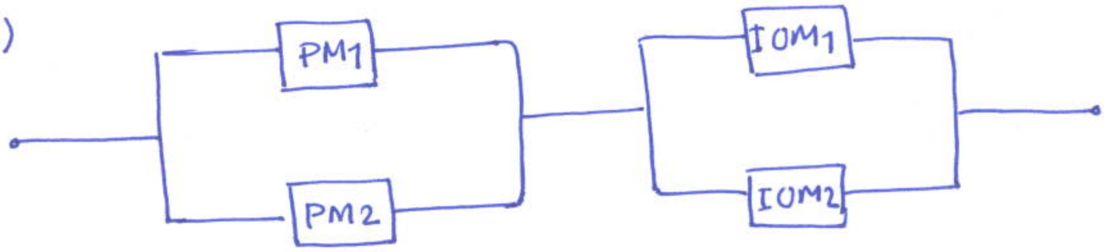
1.a) There are 8 fault containment regions:

1. PM_1 2. PM_2 3. IOM 1 4. IOM 2

5. Parallel $_{Bus}$ 1 6. Parallel $_{Bus}$ 2

7. Serial Bus 1 8. Serial Bus 2

1.b)



$$R_{node}(t) = R_{pm}(t) \cdot R_{IO}(t)$$

$$R_{PM_{1,2}} = 1 - (1 - R_{PM})^2 \qquad R_{PM} = e^{-\lambda_1 t}$$

$$= 1 - (1 - e^{-\lambda_1 t})^2$$

$$= 2e^{-\lambda_1 t} - e^{-2\lambda_1 t}$$

$$R_{I/O_{1,2}} = 1 - (1 - R_{IOM})^2 \qquad R_{IOM} = e^{-\lambda_2 t}$$

$$= 1 - (1 - e^{-\lambda_2 t})^2$$

$$= 2e^{-\lambda_2 t} - e^{-2\lambda_2 t}$$

$$R_{node}(t) = R_{PM_{1,2}}(t) * R_{IOM_{1,2}}(t)$$

$$= (2e^{-\lambda_2 t} - e^{-2\lambda_2 t}) *$$

$$(2e^{-\lambda_1 t} - e^{-2\lambda_1 t})$$

1.c)

$$\begin{tikzcd} 0 \arrow[r, bend left, "2\lambda_1 c"] \arrow[rr, bend right, "2\lambda_1(1-c)"'] & 1 \arrow[r, bend left, "\lambda_1"] & 2 \end{tikzcd}$$

Transition Matrix : $\begin{bmatrix} -2\lambda_1 & 2\lambda_1 c & 2\lambda_1(1-c) \\ 0 & -\lambda_1 & \lambda_1 \\ 0 & 0 & 0 \end{bmatrix}$

$$\begin{cases} P_0'(t) = -2\lambda_1 P_0(t) \\ P_1'(t) = 2\lambda_1 c\, P_0(t) - \lambda_1 P_1(t) \\ P_2'(t) = 2\lambda_1(1-c)\, P_0(t) + \lambda_1 P_1(t) \end{cases}$$

$$sP_0(s) - 1 = -2\lambda_1 P_0(s)$$

$$\rightarrow P_0(s) = \frac{1}{s+2\lambda_1} \xrightarrow{\mathcal{L}^{-1}} P_0(t) = e^{-2\lambda_1 t}$$

$$sP_1(s) - 0 = 2\lambda_1 c\, P_0(s) - \lambda_1 P_1(s)$$

$$\rightarrow P_1(s) = \frac{1}{s+\lambda_1} \cdot \frac{1}{s+2\lambda_2}(2\lambda_1 c)$$

$$\xrightarrow{\mathcal{L}^{-1}} P_1(t) = 2c\left(e^{-\lambda_1 t} - e^{-2\lambda_1 t}\right)$$

$$R(t) = P_0(t) + P_1(t)$$

$$= e^{-2\lambda_1 t} + 2c\left(e^{-\lambda_1 t} - e^{-2\lambda_1 t}\right)$$

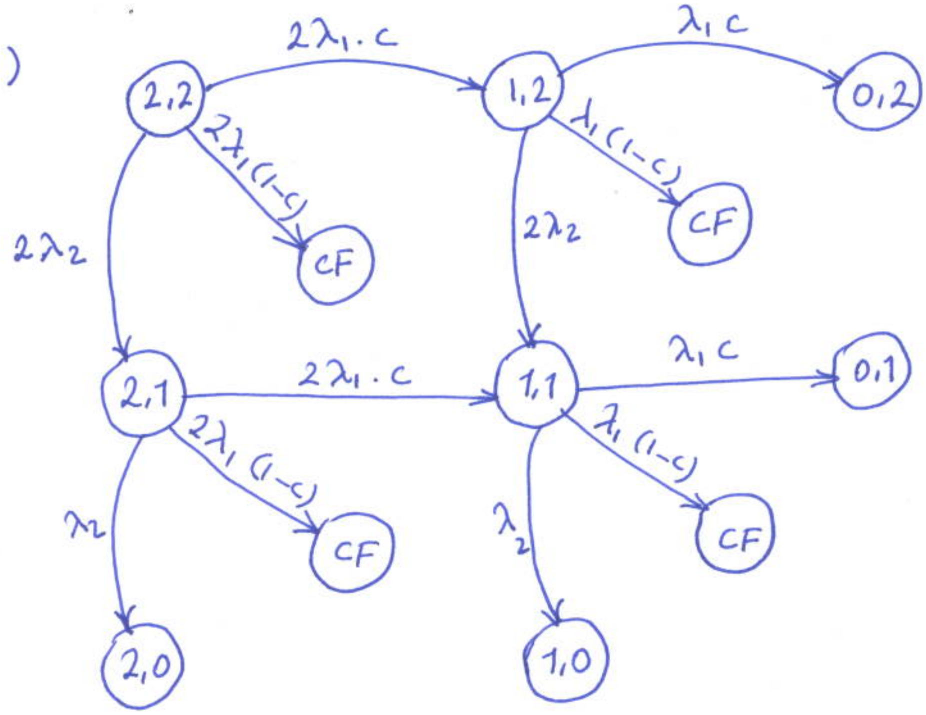
$$MTFF = \int_0^\infty R(t)\,dt$$

$$= \int_0^\infty \left(2c\, e^{-\lambda_1 t} - e^{-2\lambda_1 t} \right) dt$$

$$= \frac{-2c}{\lambda_1} e^{-\lambda_1 t} \Big]_0^\infty - \frac{-1}{2\lambda_1} e^{-2\lambda_1 t} \Big]_0^\infty$$

$$= \left(0 + \frac{2c}{\lambda_1} \right) - \left(0 - \frac{1}{2\lambda_1} \right) = \frac{2c+1}{2\lambda_1}$$

1. d)



$$1 - S(\infty) = \frac{2\lambda_1 (1-c)}{2\lambda_2 + 2\lambda_1} + \frac{2\lambda_1 c}{2\lambda_2 + 2\lambda_1} \cdot \frac{\lambda_1 (1-c)}{2\lambda_2 + \lambda_1}$$

$$+ \frac{2\lambda_2}{2\lambda_2 + 2\lambda_1} \cdot \frac{2\lambda_1 (1-c)}{\lambda_2 + 2\lambda_1}$$

$$+ \frac{2\lambda_1 c}{2\lambda_2 + 2\lambda_1} \cdot \frac{2\lambda_2}{2\lambda_2 + \lambda_1} \cdot \frac{\lambda_1 (1-c)}{\lambda_2 + \lambda_1}$$

$$+ \frac{2\lambda_2}{2\lambda_2 + 2\lambda_1} \cdot \frac{2\lambda_1 . c}{\lambda_2 + 2\lambda_1} \cdot \frac{\lambda_1 (1-c)}{\lambda_2 + \lambda_1}$$

1.d) cont.

$$S(\infty) = 1 - \left(\frac{2\lambda_1(1-c)}{2\lambda_2 + 2\lambda_1} + \frac{2\lambda_1 c}{2\lambda_2 + 2\lambda_1} \cdot \frac{\lambda_1(1-c)}{2\lambda_2 + \lambda_1} \right.$$

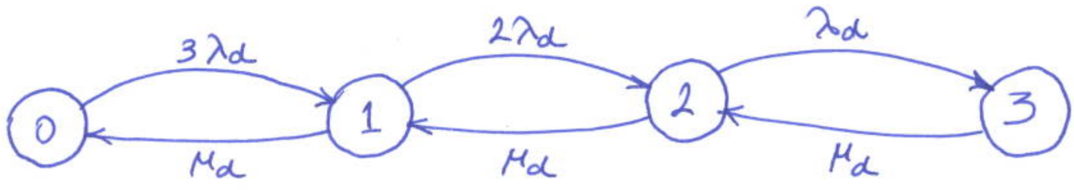
Steady Sate Safety

$$+ \frac{2\lambda_2}{2\lambda_2 + 2\lambda_1} \cdot \frac{2\lambda_1(1-c)}{\lambda_2 + 2\lambda_1}$$

$$+ \frac{2\lambda_1 c}{2\lambda_2 + 2\lambda_1} \cdot \frac{2\lambda_2}{2\lambda_2 + \lambda_1} \cdot \frac{\lambda_1(1-c)}{\lambda_2 + 2\lambda_1}$$

$$\left. + \frac{2\lambda_2}{2\lambda_2 + 2\lambda_1} \cdot \frac{2\lambda_1 c}{\lambda_2 + 2\lambda_1} \cdot \frac{\lambda_1(1-c)}{\lambda_2 + \lambda_1} \right)$$

2)



(Birth & Death Process)

$$\Pi_1 = \frac{3\lambda_d}{\mu_d}\Pi_0$$

$$\Pi_2 = \frac{2\lambda_d}{\mu_d}\Pi_1 = \frac{2\lambda_d}{\mu_d}\cdot\frac{3\lambda_d}{\mu_d}\Pi_0 = \frac{6\lambda_d^2}{\mu_d^2}\Pi_0$$

$$\Pi_3 = \frac{\lambda_d}{\mu_d}\Pi_2 = \frac{\lambda_d}{\mu_d}\Pi_1 = \frac{6\lambda_d^3}{\mu_d^3}\Pi_0$$

$$\Pi_0 + \Pi_1 + \Pi_2 + \Pi_3 = 1$$

$$\Pi_0\left(1 + \frac{3\lambda_d}{\mu_d} + \frac{6\lambda_d^2}{\mu_d^2} + \frac{6\lambda_d^3}{\mu_d^3}\right) = 1$$

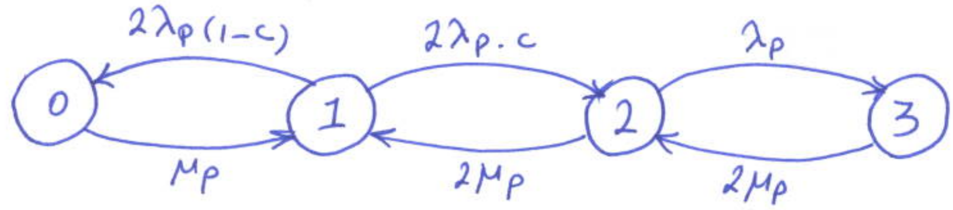
$$\Pi_0 = \frac{\mu_d^3}{\mu_d^3 + 3\lambda_d\mu_d^2 + 6\lambda_d^2\mu_d + 6\lambda_d^3}$$

$$A_{disks} = \Pi_0 + \Pi_1 + \Pi_2$$

$$= \frac{\mu_d^3}{\mu_d^3 + 3\lambda_d\mu_d^2 + 6\lambda_d^2\mu_d + 6\lambda_d^3}\left(1 + \frac{3\lambda_d}{\mu_d} + \frac{6\lambda_d^2}{\mu_d^2}\right)$$

$$= \frac{\mu_d^3 + \mu_d^2\cdot 3\lambda_d + \mu_d\cdot 6\lambda_d^2}{\mu_d^3 + 3\lambda_d\mu_d^2 + 6\lambda_d^2\mu_d + 6\lambda_d^3}$$

2) cont.



Birth and Death Process

$$\Pi_1 = \frac{\mu_p}{2\lambda_p(1-c)}\,\Pi_0$$

$$\Pi_2 = \frac{2\lambda_p . c}{2\mu_p}\,\Pi_1 = \frac{2\lambda_p . c}{2\mu_p}\cdot\frac{\mu_p}{2\lambda_p(1-c)}\,\Pi_0$$

$$\Pi_3 = \frac{\lambda_p}{2\mu_p}\,\Pi_2 = \frac{\lambda_p}{2\mu_p}\cdot\frac{2\lambda_p . c}{2\mu_p}\cdot\frac{\mu_p}{2\lambda_p(1-c)}\,\Pi_0$$

$$\Pi_0 + \Pi_1 + \Pi_2 + \Pi_3 = 1$$

$$\Pi_0\left(1 + \frac{\mu_p}{2\lambda_p(1-c)} + \frac{c}{2(1-c)} + \frac{\lambda_p . c}{4\mu_p(1-c)}\right) = 1$$

$$\Pi_0\left(\frac{4\lambda_p(1-c) + 2\mu_p + 2\lambda_p . c + \lambda_p . c}{4\lambda_p(1-c)}\right) = 1$$

$$\Pi_0 = \frac{4\lambda_p(1-c)}{4\lambda_p - \lambda_p . c + 2\mu_p}$$

$$A_{processors} = \Pi_1 + \Pi_2$$

$$= \left(\frac{4\lambda_p(1-c)}{4\lambda_p - \lambda_p . c + 2\mu_p}\right)\left(\frac{\mu_p}{2\lambda_p(1-c)} + \frac{c}{2(1-c)}\right)$$

$$= \frac{2\mu_p}{4\lambda_p - \lambda_p . c + 2\mu_p} + \frac{2\lambda_p . c}{4\lambda_p - \lambda_p . c + 2\mu_p}$$

2) cont.

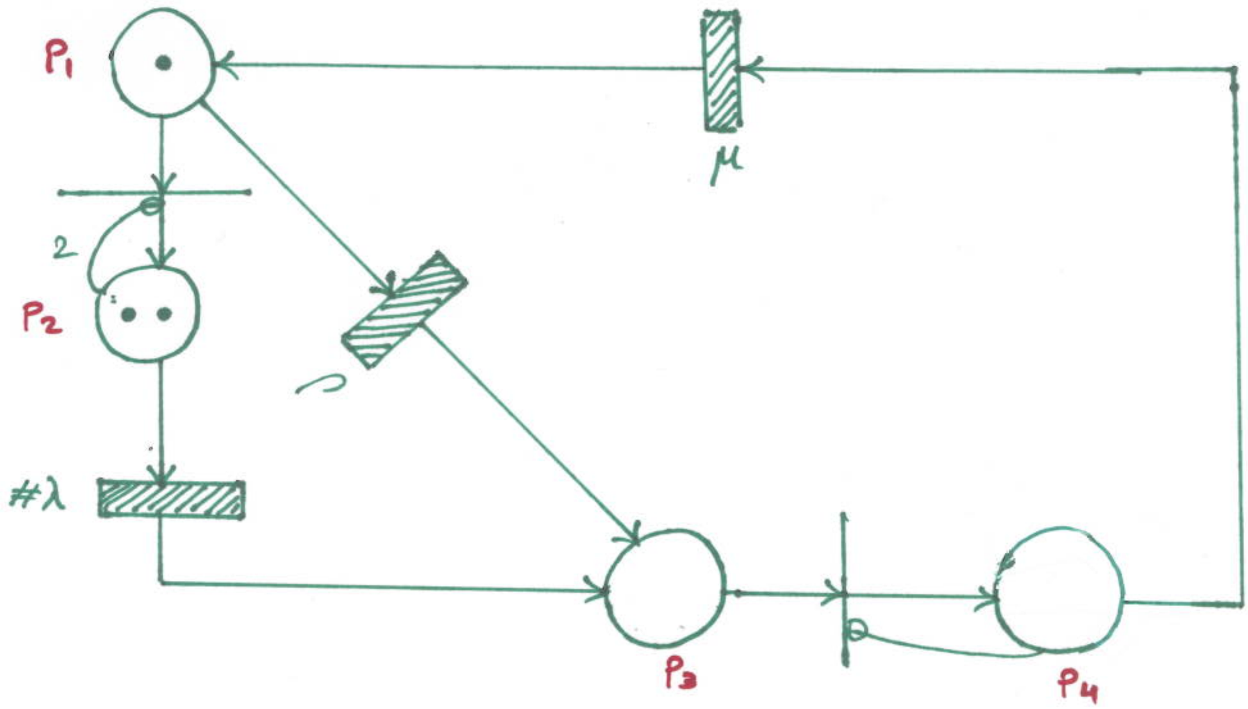
$$\Pi_{system} = \Pi_{disks} * \Pi_{processors}$$

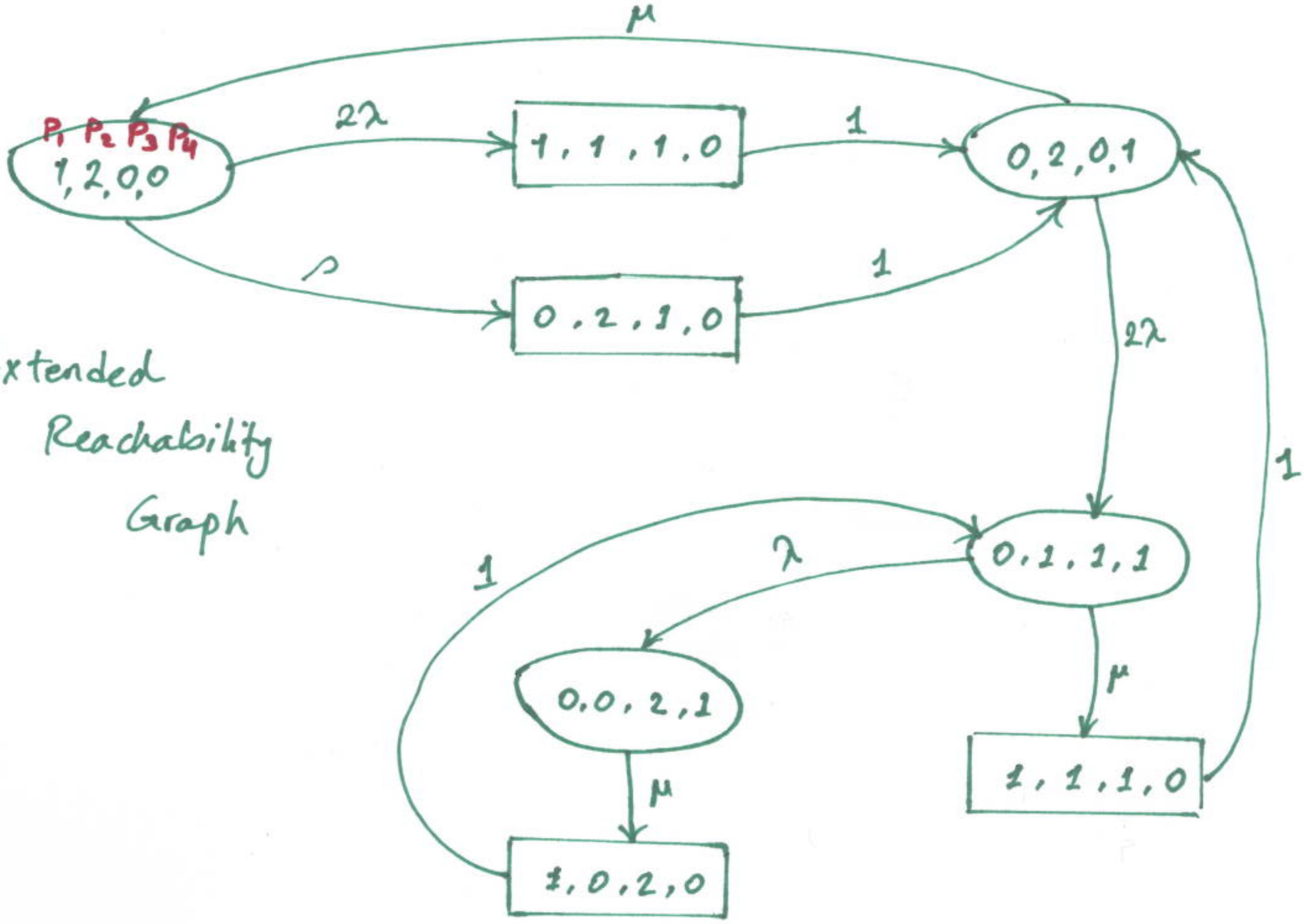
$$= \frac{\mu_d^3 + \mu_d^2 \cdot 3\lambda_d + \mu_d \cdot 6\lambda_d^2}{\mu_d^3 + 3\lambda_d \mu_d^2 + 6\lambda_d^2 \mu_d + 6\lambda_d^3}$$

$$* \left(\frac{2\mu_p}{4\lambda_p - \lambda_p \cdot c + 2\mu_p} + \frac{2\lambda_p \cdot c}{4\lambda_p - \lambda_p \cdot c + 2\mu_p} \right)$$

3)

GSPN





Extended Reachability Graph