Exam in EDA122(Chalmers) and DIT062(GU), Manday, January 10,2011, 14:00 - 18:00

Problem 1

1.a

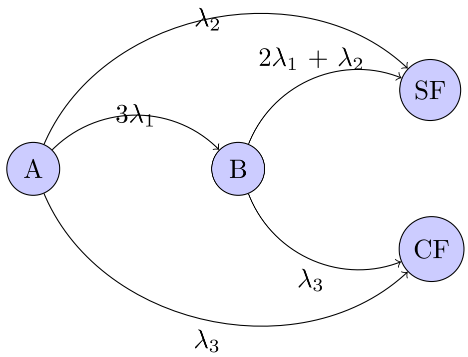


Figure 1: Markov chain

1.b

Steady-state safety

$$S(\infty) = \frac{\lambda_2}{3\lambda_1 + \lambda_2 + \lambda_3} + \frac{3\lambda_1}{3\lambda_1 + \lambda_2 + \lambda_3} * \frac{2\lambda_1 + \lambda_2}{2\lambda_1 + \lambda_2 + \lambda_3}$$

1.c

$$\begin{aligned} P'(t) &= P(t)Q \\ P(0) &= \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \\ Q &= \begin{bmatrix} -3\lambda_1 - \lambda_2 - \lambda_3 & 3\lambda_1 & \lambda_2 & \lambda_3 \\ 0 & -2\lambda_1 - \lambda_2 - \lambda_3 & 2\lambda_1 + \lambda_2 & \lambda_3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{aligned}$$

Laplace transform:

$$\mathcal{L}\{P'(t) = P(t)Q\} \Rightarrow sP(s) - P(0) = P(s)Q$$

$$\begin{cases} sP_A - 1 = -(3\lambda_1 + \lambda_2 + \lambda_3)P_A \\ sP_B = 3\lambda_1 P_A - (2\lambda_1 + \lambda_2 + \lambda_3)P_B \\ sP_{SF} = (\lambda_2)P_A - (2\lambda_1 + \lambda_2)P_B \end{cases}$$

$$\begin{aligned} P_A(t) &= e^{-(3\lambda_1+\lambda_2+\lambda_3)t} \\ P_B(t) &= 3e^{-(2\lambda_1+\lambda_2+\lambda_3)t} - 3e^{-(3\lambda_1+\lambda_2+\lambda_3)t} \\ \Rightarrow R(t) &= P_A(t) + P_B(t) \\ R(t) &= 3e^{-(2\lambda_1+\lambda_2+\lambda_3)t} - 2e^{-(3\lambda_1+\lambda_2+\lambda_3)t} \end{aligned}$$

Problem 2

Processors and disks can be considered as two independent subsystems of the system, so that their availabilities can be calculated separately. The availability of the whole system is the production of the availability of processors and disks.

The processors are modelled using the Markov-model shown in Figure 2.

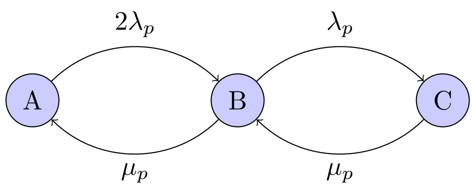


Figure 2: Markov chain for Processors

The disks are modelled using the Markov-model shown in Figure 3.

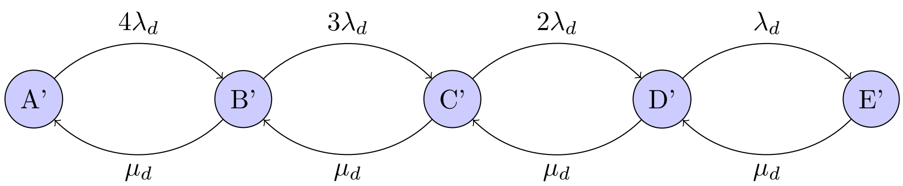


Figure 3: Markov chain for Disks

This is a birth and death process (see, e.g., Mathematics Handbook, pp. 440-441). We can use the following formulas to calculate the steady state probability for state k.

$$\begin{aligned} \Pi_k &= \frac{\lambda_{k-1}}{\mu_k}\Pi_{k-1} \\ \sum_k \Pi_k &= 1 \end{aligned}$$

Using the formulas for a birth and death process, we get(for processors):

$$\begin{cases} \Pi_B &= \frac{2\lambda_p}{\mu_p}\Pi_A \\ \Pi_C &= \frac{\lambda_p}{\mu_p}\Pi_B \\ \Pi_A + \Pi_B + \Pi_C &= 1 \end{cases}$$

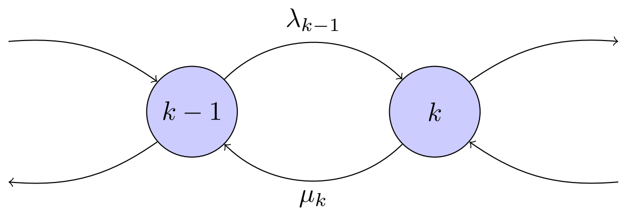


Figure 4: General birth and death process

$$\begin{aligned}
\Pi_A &= \frac{\mu_p^2}{\mu_p^2 + 2\lambda_p\mu_p + 2\lambda_p^2} \\
\Pi_B &= \frac{2\lambda_p\mu_p}{\mu_p^2 + 2\lambda_p\mu_p + 2\lambda_p^2} \\
\Pi_{Processors} &= \Pi_A + \Pi_B \\
\Pi_{Processors} &= \frac{\mu_p^2 + 2\lambda_p\mu_p}{\mu_p^2 + 2\lambda_p\mu_p + 2\lambda_p^2}
\end{aligned}$$

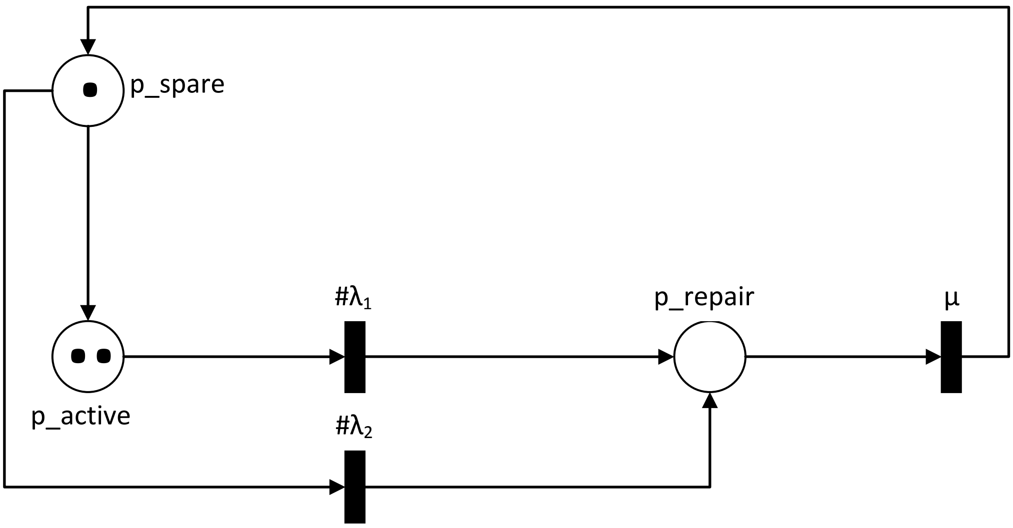
Using the formulas for a birth and death process, we get(for disks):

$$\begin{cases}
\Pi_{B'} &= \frac{4\lambda_d}{\mu_d}\Pi_{A'} \\
\Pi_{C'} &= \frac{3\lambda_d}{\mu_d}\Pi_{B'} = \frac{12\lambda_d^2}{\mu_d^2}\Pi_{A'} \\
\Pi_{D'} &= \frac{2\lambda_d}{\mu_d}\Pi_{C'} = \frac{24\lambda_d^3}{\mu_d^3}\Pi_{A'} \\
\Pi_{E'} &= \frac{\lambda_d}{\mu_d}\Pi_{D'} = \frac{24\lambda_d^4}{\mu_d^4}\Pi_{A'} \\
\Pi_{A'} + \Pi_{B'} + \Pi_{C'} + \Pi_{D'} + \Pi_{E'} &= 1
\end{cases}$$

$$\Pi_{A'} = \frac{\mu_d^4}{\mu_d^4 + 2\lambda_d\mu_d^3 + 12\lambda_d^2\mu_d^2 + 24\lambda_d^3\mu_d + 24\lambda_d^4}$$

$$\begin{aligned}
\Pi_{Disks} &= \Pi_{A'} + \Pi_{B'} + \Pi_{C'} + \Pi_{D'} \\
\Pi_{System} &= \Pi_{Processors} * \Pi_{Disks}
\end{aligned}$$

3.a



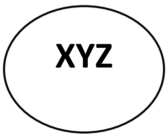
3.b

(#p_active , # p_spare, #p_repair)

Marking for system unavailability:

(0 , 0 , 3)

3.c



X: # of tokens in p_active

Y: # of tokens in p_spare

Z: # of tokens in p_repair