Solutions to April 18, 2015, exam
Mixed-Signal Systems (DAT116)

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April 20, 2015

The following are suggested solutions to the problems. It must be noted that other solutions may be possible, and that approximate solutions will give at least partial credit as long as the approximations are reasonable (just as in real life!).

1. (a) We must first estimate the magnitudes of the possible errors.
   - With the 21-MHz clock, the worst-case aliased signal is at $f_S - f_B = 21 - 7 = 14$ MHz, that is, at twice the highest frequency in the band of interest. A third-order anti-aliasing filter with its cutoff frequency at the band edge will suppress the 14-MHz signal by $3 \times 6 = 18$ dB, so the original 20-dB difference ($100\times$) will be improved to $20 + 18 = 38$ dB by the filter.
   - For the 35-MHz clock, the worst-case aliased signal is at $35 - 7 = 28$ MHz, a factor of 4 above the highest frequency of interest; thus there is another 18dB of attenuation, for a total of 36 dB. The total difference will then be $36 + 20 = 56$ dB.
   - The jitter error for the 35-MHz case is given by $SNR_{ji} = -20 \cdot \log(\Delta t \cdot \omega_{in})$ [Maloberti, eq. 1.8]; as $\omega_{in} = 2 \cdot \pi \cdot 7 \cdot 10^6 \approx 4.4 \cdot 10^7$, the error power will be 47.1 dB below the signal.

   So the faster clock will allow the better SNDR, despite its significant jitter. A steeper filter would be needed for the slower clock to be preferable.

   (b) The peak SNDR in the chosen case is 47.1 dB. Since $(47.1 - 1.76)/6.02 = 7.5$, an 8-bit converter should be chosen in order to keep the quantization noise below the jitter noise.

   (c) The sample power limit is given by a well-known equation: $P_S = 12 \cdot kT \cdot f_S \cdot 2^{2N}$. With $N = 8$ and $T = 300$, $P_S \approx 1.14 \cdot 10^{-7}$ W. A complete practical converter might need to consume roughly 200 times that power, or 23 μW.

2. (a) The signal at the input of the quantizer can be viewed as the sum of the input signal, X, and a filtered version of the difference of the input signal and the output signal, Y. Thus, in the z domain with F being the filter transfer function:

   $$Y = X + F \cdot (X - Y) = X + F \cdot X - F \cdot Y$$

   Rearrangement of terms gives:

   $$STF = \frac{Y}{X} = \frac{1 + F}{1 + F}$$

   so the STF will be 1, regardless of the exact filter transfer function F. (Note that the direct path from input to quantizer ensures there is not even a delay.)

(b) To derive the NTF, we let $X = 0$; then:

$$Y = Q - F \cdot Q = Q \cdot (1 - F)$$

$$NTF = \frac{Y}{Q} = 1 - F$$

where Q is the quantization error. Here, it actually matters what F is.
The squares in the figure each have a pole at $z = 1$, so at DC, so are integrators (multiply numerator and denominator with z^{-1} to get the more familiar form $z^{-1}/(1 - z^{-1})$). The exact placement of the filter zeroes is determined by the weight coefficients and may be calculated to be

$$z_{1,2} = 0.8187 \pm 0.1310i$$

which fully determines the NTF.

(c) Maloberti's Equation 7.10 and Figure 7.3 suggest that an SNDR of 50 dB may be reached at an OSR of 10 with a third-order loop. (This first-order estimate disregards the influence of the loop zeroes, which is exemplified in Figure 7.4.)

3. (a) Let

$$x(t) = A \sin \omega t$$

and then

$$y(t) = x(t) - \alpha(x(t))^3 = A \sin \omega t - \alpha A^3 \sin^3 \omega t$$

Since

$$\sin 3\phi = 3 \sin \phi - 4 \sin^3 \phi$$

we have

$$y(t) = A \sin \omega t - \alpha A^3 \cdot \frac{1}{4}(3 \sin \omega t - \sin 3\omega t) = A(1 - \frac{3}{4}\alpha A^2) \sin \omega t + \frac{1}{4}\alpha A^3 \sin 3\omega t$$

As expected, the amplitude of the fundamental tone (at ω) does not follow the input amplitude at increased A.
A 1-dB reduction corresponds to a power reduction by a factor of $10^{-0.1} \approx 0.8$ and an amplitude reduction of $\sqrt{10^{-0.1}} \approx 0.9$. Thus, the compression point is reached when

$$1 - \frac{3}{4}\alpha A^2 = 0.9$$

so

$$A^2 = \frac{0.4}{3\alpha}$$

or

$$A = \sqrt{\frac{0.4}{3\alpha}}$$

(b) Since $\alpha = 0.1$,

$$A^2 = \frac{0.4}{3 \cdot 0.1} = \frac{0.4}{0.3} \approx 1.33$$

So, the compression point is $10 \cdot \log_{10}(1.33) \approx 1.25$ dB above the amplitude $A = 1$.

(c) The intercept point is the input level when the third harmonic would have the same power (and therefore amplitude) as the undistorted fundamental tone would have:

$$A = \frac{1}{4}\alpha A^3$$

So, with $\alpha = 0.1$,

$$A^2 = \frac{4}{\alpha} = 40$$

This is $10 \cdot \log_{10}(40) \approx 16.0$ dB above $A = 1$, and therefore $16.0 - 1.25 = 14.8$ dB above the compression point.

4. (a) The sample capacitance limit may be straightforwardly calculated by this formula:

$$C_S = \frac{12kT}{V_{FS}^2} 2^{2N}$$

With $T = 373$ K (100°) and $V_{FS} = 1$ V, $C_S \approx 3.7$ fF. This limit applies to the sample capacitance, that is, to the sum of *all* the $C/2^i$ in the conceptual schematic (all these capacitances are connected in parallel when the signal is sampled!). Thus, the noise requirements limit C to about half of this value, or 1.8 fF.

(Actually, the matching limit is much more stringent, as seen in the next subproblem; V_{FS} may take a different value than 1 V; and the thermal noise that limits the ENOB is generated in the comparator input stage rather than in the sampler. . .)

(b) With the smallest capacitance at 2 fF, each of the four capacitor banks has a capacitance of $2 \cdot \sum_{i=0}^{7} i = 510$ fF. With differential signalling and a 1-V supply, and with resetting the inputs before each conversion, the voltage swing across each capacitance will be at most 500 mV. The total energy expended is then $4 \cdot 510 \cdot 0.5^2 = 510$ fJ.

(c) The third-least-significant capacitor is now minimum-sized, and the more-significant capacitors use conventional binary scaling. Thus the total capacitance is close to 1/4 of the value for the previous case, as is the power.

One extra benefit is that the input capacitance is also reduced; in fact, the Kull converters use no input signal buffer, which makes sense since the converter input capacitance is on the order of that of a bonding pad (roughly 100 fF); the external circuit that drives the pad also directly drives the converter input.