Solutions to Jan 15, 2015, exam Mixed-Signal Systems (DAT116)

Lars Svensson, Lena Peterson

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The following are suggested solutions to the problems. It must be noted that other solutions may be possible, and that approximate solutions will give at least partial credit as long as the approximations are reasonable (just as in real life!).

1. (a) According to Nyquist, the clock frequency / sample rate needs to be at least twice the signal bandwidth, i.e., 14 MHz. The standard SNR formula, $6.02 \times N + 1.76$, suggests that 8 bits should be enough to reach 48 dB.

   (b) The quantization noise is less than 3 dB below the limit given by the specification, so we cannot just set the jitter power equal to that of the quantization noise. The latter is $6.02 \times 8 + 1.76 = 49.92 \approx 50$ dB below the peak signal power. As we showed during the lab course, this means that the jitter error must be 4.3 dB below the 48-dB limit, so at 52.3 dB below the signal power.

   The power of the jitter error for a single-sine-wave input signal of amplitude A and frequency f is given by:

   $$\frac{A^2(2\pi f)^2}{2}\Delta t_{RMS}^2$$

   The power of the full-scale signal is $A^2/2$, so:

   $$\Delta t_{RMS}^2 < \frac{10^{-52.3/10}}{(2\pi f)^2} \approx \frac{5.9 \cdot 10^{-6}}{(44 \cdot 10^6)^2} \approx \frac{5.9 \cdot 10^{-6}}{2 \cdot 10^{15}} \approx 30 \cdot 10^{-22}$$

   Thus

   $$\Delta t_{RMS} < 54\text{ps}$$

   (c) Assuming a low-pass filter with no zeros in the stopband, a 3rd-order filter suppresses frequencies beyond the cutoff by 18 dB per octave. We wish the attenuation in all of the filter stopband to be at least 48 dB (in order to suppress a full-range sinewave below the power of the other noise components). This level of suppression requires $48/18 = 2.6$ octaves, so a factor of $2^{2.6} \approx 6.06$ in frequency. Thus, if the sample rate is chosen at 3.5 times the bandwidth (at 24.5 MHz), anything aliased into the band-of-interest will be sufficiently suppressed before sampling. (Signals above the band of interest but below half the sample rate, in the range between 7 MHz to 12.25 MHz, may be removed with digital filters after quantization).

2. (a) The gain error in this type of amplifier is mainly due to 1) the finite open-loop gain of the operational amplifier, and 2) the imperfect matching of the resistors in the feedback network.

   Due to the low-pass character of the gain curve, the open-loop gain is at its lowest (and thus the discrepancy is at its worst) for the highest frequency of interest: 1 kHz. At 1 kHz, the OP27 open-loop gain is approximately 80 dB, i.e., 10000 times, according to the curve in Figure 2. The discrepancy is given by:

$$D = \frac{A\beta}{1 + A\beta}$$

   and the gain error is $1 - D$.

   Then, with a closed-loop gain of 100 (40 dB), the error just due to the finite op-amp gain is already 1%. For an overall gain error of 1%, the resistors would then have to be perfectly matched, which is not realistic. The two-stage solution is clearly the preferred one.

   (b) The worst case for the error is for the highest closed-loop gain, that is, when $A_{CL} = 10$ for each of the two identical stages. The error ϵ is then equal for each stage. In the worst case, both errors have the same sign: $(1 + \epsilon)(1 + \epsilon) = 1 + 2\epsilon + \epsilon^2$, where the square term can be ignored. Thus we may assign half of the allowable error to each stage.

   When $A_{CL} = 10$, the gain-related error is 0.1% in each stage. The allowed matching-related error in each stage is then 0.4%.

   (c) The bandwidth of the new amplifier is higher by a factor of 2, so the gain-related error is approximately halved: the two-stage solution suffers from 0.05% per stage, leaving 0.45% for mismatch; and the one-stage solution suffers from 0.5%, leaving 0.5% for mismatch. Clearly the matching requirements are similar in the two cases.

   For 0 dB closed-loop gain ($A_{CL} = 1$) we connect the negative input directly to the output, so we only need to provide resistors for the other cases.

   In the first design, we need four resistors: two each of R1 and R2 = 10R1. In the second design, we need three resistors with values R1, 10R1, and 100R1.

   Resistance is given by the ratio of the resistor length to the width, with the width chosen for matching. Thus, the first design needs a total resistor length of $2 \cdot (1 + 10) \cdot W_1 = 22 \cdot W_1$, and the second one needs $(1 + 10 + 100) \cdot W_2 = 111 \cdot W_2$. Matching requirements let us set $W_2/W_1 = 0.45/0.5$. Thus, discounting the complications of actually laying out the resistors, it appears that the resistor area of the second design is $(111/22) \cdot (0.45/0.5) = 4.5$ times larger than for the first one.

3. (a) According to the well-known formula, the quantization noise can be estimated to be $6.02 \times 10 + 1.76 \approx 62$ dB below a full-scale sine input.

   With gains of 1 and $1 - \epsilon$ in the two channels, a signal of amplitude $\epsilon/2$ will appear to be overlaid on the real signal, whose amplitude will appear to have been reduced to $1 - \epsilon/2$. The power of the overlaid signal can be approximated as $1/2 \cdot (\epsilon/2)^2 = \epsilon^2/8$, a factor of $\epsilon^2/4$ smaller than the full-scale sine input.

   It is clear that a gain error of 0.001 (corresponding to –60 dB) will contribute about as much to the SNDR as the quantization noise does. (This agrees with intuition:

the error is about 1 part in 1000, as is the quantization error, since $2^{10} = 1024$. This level of matching is typically not reachable by simple duplication of the circuits from one path to the next!)

(b) If one path is affected by a delay τ, the appearance is of an alternating delay of $\pm\tau/2$. The corresponding amplitude error depends on the maximum time derivative of the input signal, which for a full-scale sine wave at f_B is $2\pi f_B$. For an error on the order of 1 part in 1000, $(\tau/2) \cdot (2\pi f_B) = \pi\tau f_B = 0.002$. Thus, the skew must be at most 1/1570 of the cycle time of the highest-frequency signal (again on par with what intuition would suggest).

(c) The overlaid signal will now be of the frequencies $f_S/3$ and $2f_S/3$; the spurs will be centered around those frequencies in agreement with sums and differences with the signal frequency.

4. (a) According to the Maloberti formula (6.25) for a second-order loop, $SNDR_{peak} = 6.02 \times 1 + 1.76 - 12.09 + 6 \times 15.05 = 86$ dB.

(b) The length of the impulse-response "box", T, determines the positions of the transfer-function zeros. Clearly we prefer not to have any zeros in the band of interest, so $T < 1/f_B$, where f_B is the width of the band of interest. As indicated in the given diagram, the magnitude value at $1/2T$ is 0.636[1] which corresponds to 3.9 dB of attentuation. To have less than 3 dB of attenuation at f_B (which may be a reasonable design point), T would need to be somewhat smaller than $1/2f_B$; $T = 0.44/f_B$ gives 2.96 dB. If the OSR is 64, a period at f_B is 128 samples, so T is $0.44 \cdot 128 \approx 56$.

The rolloff of the filter is bounded by the $1/f$ factor of the sinc function; thus the reduction is never worse than for a first-order filter, at 20 dB per decade. The sine factor, which turns up squared in the power spectrum of the filtered signal, varies between 0 and 1, with an average value of 0.5. If the noise power distribution across frequencies is uncorrelated with the transmission zeros, we should expect the total noise power to be approximately 3 dB lower than for the first-order filter without the sine factor.

It must be noted that there are much better ways to select the tap currents for the filter; but FIR filter design is outside the scope of this course.

(c) A single extra pole is always placed on the negative real axis in the s plane; its distance from the origin is the same as that of the single-pole-filter cutoff frequency on the imaginary axis. The single-pole cutoff should be close to the edge of the band of interest (the best position would depend on the DT filter). Thus, the R and C of the integrator/buffer should roughly be related as follows:

$$R \cdot C = \tau = \frac{1}{2\pi \times 2 \cdot 10^4} = 8 \cdot 10^{-6}$$

With the extra pole added to the first-order characteristics of the DT filter, the overall noise suppression rolloff will match that of a second-order filter, which is what would be needed for a second-order noise-shaping loop. Again, the tap currents may be chosen for additional suppression.

[1] Since $\frac{\sin(\pi/2)}{\pi/2} = \frac{2}{\pi} \approx 0.636$