Inst.: Data- och informationsteknik
Kursnamn: Logic in Computer Science
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Kurs: DIT201/DAT060
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All answers and solutions must be carefully motivated!
total 60; $\geq$28: 3, $\geq$38: 4, $\geq$50: 5
total 60; $\geq$28: G, $\geq$42: VG

All answers **must** be carefully motivated.

1. Give proofs in natural deduction of the following sequents:
   (a) $(p \vee q), \neg q \vdash p$ (3p)
   (b) $\neg(p \vee q) \vdash \neg p$ (3p)
   (c) $p \vee (q \to r) \vdash (p \vee q) \to (p \vee r)$ (3p)

2. Let P, S and M be unary predicates and R a binary predicate. Decide for each of the sequents below whether they are valid or not, i.e., give a proof in natural deduction or a counter-model. (12p)
   (a) $\exists x\,(P(x) \wedge \neg M(x)), \exists y\,(M(y) \wedge \neg S(y)) \vdash \exists z\,(P(z) \wedge \neg S(z))$
   (b) $\forall x \neg R(x,x) \vdash \forall x\,\forall y\,(R(x,y) \to \neg R(y,x))$
   (c) $\forall x\,\forall y\,(R(x,y) \to \neg R(y,x)) \vdash \forall z\,\neg R(z,z)$
   (d) $\vdash \forall x \exists y\, R(x,y) \vee \exists x \forall y\, \neg R(x,y)$

3. We consider the following language: we have one binary predicate symbol R, a unary function symbol f, and a constant c.
   (a) Give two examples of a model of this language. (3p)
   (b) Explain why the formula $(\forall x\, R(f(x),x)) \to R(c,c)$ is *not* provable. (2p)

4. Give all truth values that make the following formula true $(p \to q) \vee ((r \to \neg q) \wedge \neg p)$ (2p)

5. Let P, Q, and R be unary predicate symbols, f and g unary function symbols. Give proofs in natural deduction of the following sequents:
   (a) $\forall x\,(P(x) \to (Q(x) \vee R(x))), \neg\exists x\,(P(x) \wedge R(x)) \vdash \forall x\,(P(x) \to Q(x))$ (4p)
   (b) $\forall x\,(f(g(x)) = x) \vdash \forall x \exists y\,(x = f(y))$ (4p)

6. Let $Pow(A)$ be the set of subsets of a given finite set A and $F : Pow(A) \to Pow(A)$ be a function such that $F(X) \supseteq F(Y)$ if $X \subseteq Y$. Explain why there exists a subset Z of A such that $F(F(Z)) = Z$. (3p)

7. Consider the transition system $\mathcal{M} = (S, \to, L)$ where the states are $S = \{s_0, s_1, s_2, s_3\}$, the transitions are $s_0 \to s_0, s_0 \to s_1, s_0 \to s_2, s_1 \to s_1, s_1 \to s_2, s_2 \to s_3$, and $s_3 \to s_2$, and the labeling function is given by $L(s_0) = \{p\}, L(s_2) = \{q\}$, and $L(s_1) = L(s_3) = \{r\}$.

(a) Do we have $\mathcal{M}, s_0 \models \mathrm{E}[p \,\mathrm{U}\, r]$? (2p)

(b) Which are the states s that satisfy the CTL formula $\mathrm{AF}\, q$ (i.e., where $\mathcal{M}, s \models \mathrm{AF}\, q$)? (2p)

(c) Is the LTL formula $\mathrm{G}\,\mathrm{F}\, r \rightarrow \mathrm{X}(q \vee r)$ satisfied on all paths? (2p)

(d) Explain why the LTL formula $\mathrm{G}\,\neg p \rightarrow \mathrm{G}\,\mathrm{F}\, r$ is satisfied on every path in $\mathcal{M}$. (2p)

8. Are the following LTL formula valid, i.e., satisfied on all paths of all transition systems? (9p)

$$(\mathrm{F}\, p \wedge \mathrm{F}\, q) \rightarrow ((\mathrm{F}(p \wedge \mathrm{F}\, q)) \vee \mathrm{F}(q \wedge \mathrm{F}\, p))$$

$$\mathrm{F}(p \rightarrow \mathrm{X}\, p)$$

$$(\mathrm{G}(p \rightarrow \mathrm{X}\, p)) \rightarrow (p \rightarrow \mathrm{G}\, p)$$

9. We fix a language with a relation symbol R. Give a model which validates all the following formulae (4p)

$$\forall x\ \neg R(x,x) \qquad \forall x\ y\ z\ (R(x,y) \wedge R(y,z)) \rightarrow R(x,z),$$

$$\forall x\ \exists y\ (R(x,y) \wedge \forall z\ (R(x,z) \rightarrow \neg R(z,y)))$$

Good Luck!

Jan and Thierry