Inst.: Data- och Informationsteknik
Kursnamn: Logic in Computer Science
Examinator: Thierry Coquand
Kurs: DIT200/DAT060
Datum: 2012-08-22 No help documents
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All answers and solutions must be carefully motivated!
total 30; $\geq$14: 3, $\geq$19: 4, $\geq$25: 5
total 30; $\geq$14: G, $\geq$21: VG

1. Give a proof in natural deduction of

   a) $((p \to q) \wedge (p \to r)) \to (p \to (q \wedge r))$ (2p)

   b) $(\exists x.(P(x) \vee Q(x))) \to ((\exists x.P(x)) \vee (\exists x.Q(x)))$ (2p)

   c) $(\forall x.\forall y.(S(y) \to F(x))) \to ((\exists y.S(y)) \to \forall x.F(x))$ (2p)

2. Say whether these consequences are true or false (if it is true give a proof, and if it is false give a model where it is not valid):

   a) $(\exists x.(P(x) \to Q(x))) \to ((\exists x.P(x)) \to (\exists y.Q(y)))$ (2p)

   b) $[(\forall x.\exists y.R(x,y)) \wedge (\forall y.\exists x.R(x,y))] \to \forall x.R(x,x)$ (2p)

3. We consider the following language: we have one unary predicate symbol P and one unary function symbol f and one constant a. Define what is a *model* for this language (2p). Give an example of a formula in this language, containing an occurence of a, f and P, which holds in some model but not in all models (1p).

4. Compute a CNF for the formula

   $(p \to q) \vee (q \wedge r)$ (1p)

   $(p \to (q \to r)) \to ((p \to q) \to r)$ (1p)

5. We consider the following CTL model $(S, \to, L)$ where S has 4 states s_0, s_1, s_2, s_3 and we have $s_0 \to s_0,\ s_0 \to s_1, s_0 \to s_3,\ s_1 \to s_1,\ s_1 \to s_2,\ s_2 \to s_0,\ s_2 \to s_3,\ s_3 \to s_0$ and $L(s_0) = \{a\},\ L(s_1) = \{a, c\},\ L(s_2) = \{b\}$ and $L(s_3) = \{b, c\}$. What are the states s for which we have?

   a) $s \models EX\ (EX\ c)$ (1p)?

   b) $s \models AG\ (EF\ b)$ (1p)

6. We consider the language with one unary predicate symbol P and one function symbol P and one constant a. We consider the formulae $\phi_1 = P(a)$ and $\phi_2 = \forall x.(P(x) \to P(f(x))$ Explain why we don't have $\phi_1, \phi_2 \vdash \forall x.P(x)$ (2p). Give a model where we have $\phi_1 \wedge \phi_2 \to \forall x.P(x)$ (2p).

Please turn the page!

7. Is the following LTL formula

$$F(\varphi) \to \varphi \vee X(X(F(\varphi)))$$

valid? Motivate your answer (2p)

8. Is the following entailment valid? (2p)

$$(p \to q) \to r,\ \neg r \wedge \neg s,\ (q \to p) \vee t,\ t \to (r \vee p) \vdash t$$

9. We consider the function $F : X \longmapsto (\{1, 3\} \cup X) \cap \{1, 4\}$ which takes as argument a subset to $\{1, 2, 3, 4\}$ and return a subset to $\{1, 2, 3, 4\}$. What is the least fixed point of F and the greatest fixed point of F (2p)? Can you give another fixed point of F (1p)?

10. Explain why $\to,\ \neg$ is a complete set of connectives (2p).

Good Luck!

Thierry and Jan