Inst.: Data- och Informationsteknik
Kursnamn: Logic in Computer Science
Examinator: Thierry Coquand
Kurs: DIT201/DAT060
Datum: 2011-10-18 No help documents
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All answers and solutions must be carefully motivated!
total 30; $\geq$14: 3, $\geq$19: 4, $\geq$25: 5
total 30; $\geq$14: G, $\geq$21: VG

1. Give a proof in natural deduction of

   a) $\vdash p \to \neg\neg\, p$ (2p)

   b) $\forall x\ (P(x) \to Q(x) \vee R(x)),\ \neg\exists x\ (P(x) \wedge R(x)) \vdash \forall x\ (P(x) \to Q(x))$ (2p)

   c) $\neg(p \to q) \vdash p$ (2p)

   If you use a derived rule (e.g. Modus-Tollens) you should justify the use of this rule by giving a derivation.

2. Explain why $\{\to,\ \neg\}$ is an adequate set of connectives (for any formula there is an equivalent formula using connectives from this set) (2p).

3. Say whether these consequences are true or false (if it is true give a proof, and if it is false give a model where it is not valid):

   a) $(\exists x\ P(x)) \to (\exists x\ Q(x)) \vdash \forall x\ (P(x) \to Q(x))$ (1p)

   b) $\forall x\ \exists y\ R(x, y) \vdash \exists x\ R(x, x)$ (1p)

   c) $\forall x\ (P(x) \to Q(x)),\ \exists x\ (P(x) \wedge R(x)) \vdash \exists x\ (Q(x) \wedge R(x))$ (1p)

   d) $\vdash (\forall x\ (P(x) \vee Q(x))) \to (\forall x\ P(x)) \vee (\forall x\ Q(x))$ (2p)

4. We consider the following CTL/LTL model $M = (S, \to, L)$ where S has 4 states s_0, s_1, s_2, s_3 and we have $s_0 \to s_0, s_0 \to s_1,\ s_1 \to s_2,\ s_2 \to s_1,\ s_2 \to s_3,\ s_3 \to s_3$ and $L(s_1) = L(s_2) = \{p\},\ L(s_3) = \{q\},\ L(s_0) = \emptyset$. Explain why the LTL formula $G(p \to F\ q)$ is not valid at all states of the model M (2p). Is there a CTL formula φ such that, for any model N, we have $N, s \models_{CTL} \varphi$ if, and only if, $N, s \models_{LTL} G(p \to Fq)$ for all states s of N (2p)?

5. Compute a CNF for the formula

   $p \to (\neg r \wedge (p \to q))$ (1p)

   $(p \wedge q \to r) \vee (p \wedge q \wedge r)$ (1p)

6. Let S be the set of natural numbers n such that $1 \leq n \leq 10$ and $P(S)$ the set of subsets of S. We consider the function $F : P(S) \to P(S)$ defined by: if $1 \in X$ then $F(X) = X - \{1\}$, and if $1 \notin X$ then $F(X) = X \cup \{1\}$. Is F monotone (1p)? Is there a fixed point of F (1p)?

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7. Is the following LTL formula valid (2p)?

$$G\ (p \to XF\ p) \land p \quad \to \quad GF\ p$$

8. Consider the following CTL model with 5 states s_0, s_1, s_2, s_3, s_4 and transition $s_0 \to s_1,\ s_1 \to s_2,\ s_2 \to s_3,\ s_3 \to s_1,\ s_0 \to s_4,\ s_4 \to s_4$ and $L(s_2) = \{p\}$ and $L(s_i) = \emptyset$ for $i \neq 2$. What is $SAT(EF\ (AF\ p))$ (2p)

9. Is the following entailment valid (2p)?

$$(q \to p) \to t,\ \neg t \land \neg s,\ (p \to q) \lor r,\ r \to (t \lor q) \vdash r$$

10. We consider the language with a predicate symbol P and a function symbol f. What is a model of this language (1p)? Explain why the following formula φ holds in all possible models (2p)

$$\varphi = \exists x\ \forall y\ (P(x) \to P(f(y)))$$

Good Luck!

Thierry, Simon and Jan