Test Exam

1. We consider the following language: we have one binary relation symbol R, and one unary function symbol f. Define what is a *model* for this language (2p). Give an example of a formula in this language which does not hold in all models (1p).

2. Compute a CNF for the following propositional formulae

   - $\neg(p \to (\neg(q \vee (\neg p \to q))))$ (1p)
   - $\neg p \wedge (q \to p) \wedge (r \to q)$ (1p)

3. Four men and four women are nominated for two positions. Exactly one man and one woman are elected. The men are A, B, C, D and the women are E, F, G, H. We know:

   - if neither A nor E won, then G won
   - if neither A nor F won, then B won
   - if neither B nor G won, then C won
   - if neither C nor F won, then E won

   Who were the two people elected (2p)?

4. Show that $\{\vee, \neg\}$ is an adequate set of connectives (2p).

5. What are the models of the following formula (2p)

$$\forall x \forall y \forall z.(x = y \vee y = z \vee x = z)$$

6. We consider the language where T is a unary predicate and R a binary predicate. Show that $\phi_1, \phi_2, \phi_3 \vdash \perp$ where ϕ_1 is $\forall x \forall y \forall z\ (R(x, y) \wedge R(y, z) \to R(x, z))$, ϕ_2 is $\forall x \exists y\ R(x, y)$ and ϕ_3 is $T(x) \leftrightarrow \forall y\ (R(x, y) \to \neg T(y))$ (3p)

7. For the language $\{0, S\}$ we consider the formulae F_1 which is $\forall x\ \neg(S(x) = 0)$, F_2 which is $\forall x \forall y\ (S(x) = S(y) \to x = y)$ and F_3 which is $\forall x\ x = 0 \vee \exists y\ (x = S(y))$. Find a model where we have an element satisfying the formula $u = S(u)$ (2p). Explain why we cannot have $F_1, F_2, F_3 \vdash \forall x \neg(S(x) = x)$ (1p)

8. Say which formula is true and which one is false (if it is true give a proof, and if it is false give a model where it is not valid):

   - $\exists x\ (\neg Q(x) \wedge P(x)) \to \forall x\ (Q(x) \to P(x))$ (1p)
   - $\forall x\ (P(x) \to Q(x) \wedge Q(x) \to P(x)) \to (\exists x\ P(x) \to \exists x\ Q(x))$ (1p)
   - $(\exists x\ P(x) \to \exists x\ Q(x)) \to \exists x\ (P(x) \to Q(x) \wedge Q(x) \to P(x))$ (1p)

9. We consider the following CTL model $(S, \to, L)$ where S has 3 states s_1, s_2, s_3 and we have $s_1 \to s_2,\ s_1 \to s_3,\ s_2 \to s_1,\ s_2 \to s_3,\ s_3 \to s_3$, and $L(s_1) = \{a, b\},\ L(s_2) = \{b, c\}$ and $L(s_3) = \{c\}$.

   Write the beginning of the unwinding tree of this system, the starting state being s_1 (1p).

   Explain why we have $s_1 \models AG(c \to EGc)$ and why we do not have $s_1 \models AG(EFb)$ (3p).